

# Are Quasimonoenergetic Annihilation Photons Enough Monoenergetic Really?

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**Abstract.** Absolute majority of photonuclear reaction cross section data have been obtained in two different type experiments using electron bremsstrahlung and quasimonoenergetic photons produced by annihilation in flight of relativistic positrons. Unfortunately there are many clear systematical disagreements both in shape and magnitude between data obtained in different experiments. Very shortly the main of them can be described as following: as a rule reaction cross sections obtained in experiments used quasimonoenergetic annihilation photons in comparison with that obtained in experiments used bremsstrahlung look like as much more smooth. The reasons of that were investigated in details on the base of all (final and intermediate) results of some typical experiments with annihilation photons. It was shown that photonuclear reaction cross sections obtained with annihilation photons are strongly over-smoothed and have real energy resolution is several (3 – 4) times worse than estimated one (calculated positron annihilation line width). Therefore for detailed study of reaction cross section structure quasimonoenergetic annihilation photons are not enough monoenergetic because the real energy resolution ( $\sim 1.3 - 1.6$  MeV) gives not to one possibility to investigate physical effects produced structures with smaller width.

# INTRODUCTION

Well-known clear disagreements<sup>1,2)</sup> in shape between photonuclear reaction cross sections obtained correspondingly in experiments used bremsstrahlung (BR) carried out primarily in Russia Moscow State University, Australia Melbourne University and some other laboratories and quasimonoenergetic photons from annihilation (QMA) in flight of relativistic positrons carried out generally at USA National Lawrence Livermore Laboratory and France Centre d'Etudes Nucleaires de Saclay have been investigated in many works<sup>3-6)</sup>. It was obtained that as a rule reaction cross sections obtained in experiments used QMA-photons in comparison with that obtained in experiments used BR-photons look like as much more smooth: there are no pronounced structures in photonuclear reaction cross sections for majority of nuclei with exception of several ones with  $A$  till about 30.

It was shown<sup>5-6)</sup> that the reason is the clear difference in the shape of effective spectrum of incident photons – apparatus function of experiment. Very briefly – that for BR-experiment is sufficiently enough localized in energy, but that for QMA-experiment not. The method of reduction<sup>5-6)</sup> was developed for effective transformation of reaction cross section from presentation at each concrete apparatus function of complicated shape with practically unknown energy resolution to that of unique Gaussian-shape with clearly determined energy resolution. It was obtained that the method of reduction give to one the possibility to obtain (to transform) data from various experiments to the same presentation and therefore to do the comparison more reliable. It was shown that more apparatus function of various experiments closer to Gaussian line systematical disagreements between cross sections became smaller. But the problem of each experiment, first of all QMA-, real initial energy resolution estimation remained open.

The main aim of this work is to investigate in details the problem of energy resolution loosing on the base of all (final and intermediate) results of some typical experiments with annihilation photons traditionally used the subtraction procedure for obtaining (not direct measurement) of the photonuclear reaction cross section.

# EFFECTIVE PHOTON SPECTRUM AND CROSS SECTION SHAPE

## Two Main Types of Photonuclear Experiments

**1) Bremsstrahlung spectrum** is continuous and therefore not direct reaction cross section is measured in experiment but only reaction yield  $Y(E_{jm}, k)$  - cross section  $\sigma(k)$  with threshold  $E_{th}$  depended on photon energy  $k$  folded with photon spectrum  $W(E_{jm}, k)$  with end-point energy  $E_{jm}$ :

$$Y(E_{jm}) = \frac{N(E_{jm})}{\varepsilon D(E_{jm})} = \alpha \int_{E_{th}}^{E_{jm}} W(E_{jm}, k) \sigma(k) dk. \quad (1)$$

The yield  $Y(E_{jm}, k)$  presents the smooth increased function of the photon spectrum end-point energy  $E_{jm}$  and each cross section resonance is reflected in the yield curvature (derivative) changing.

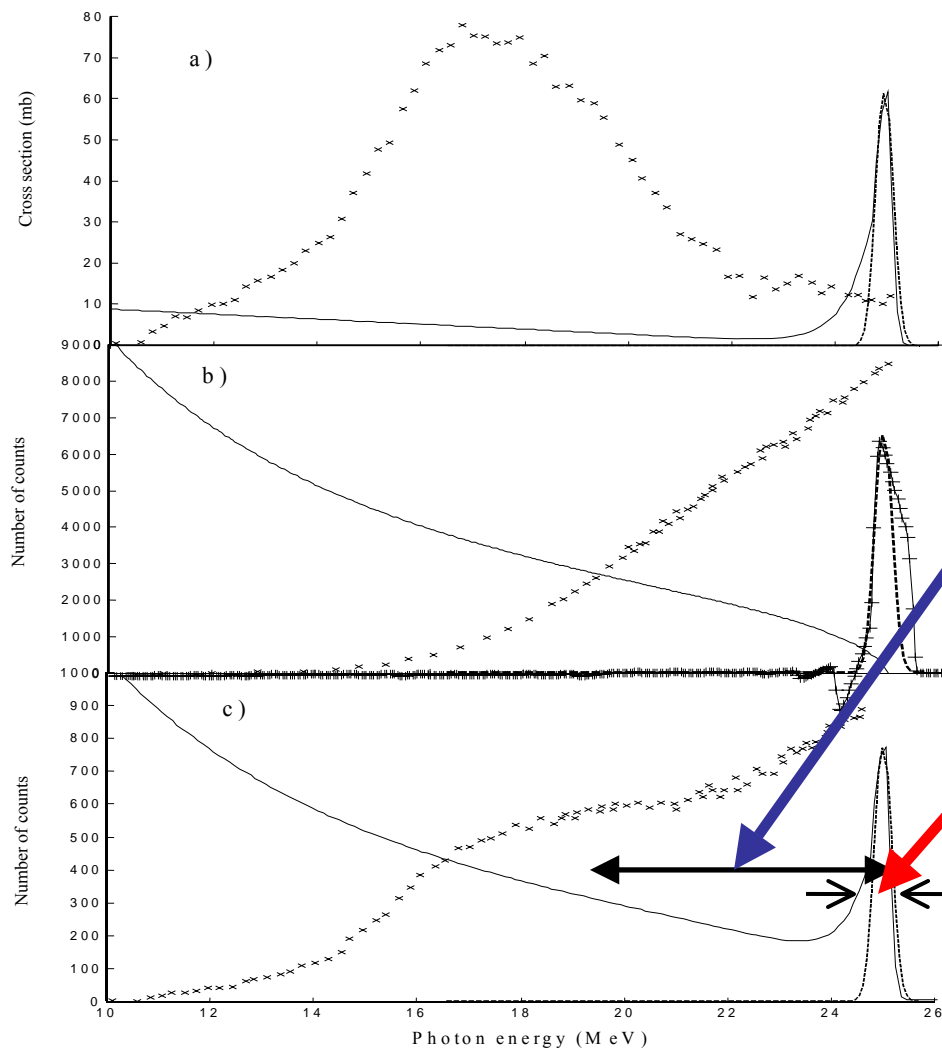
Cross section  $\sigma$  can be obtained<sup>1,2)</sup> from the yield  $Y$  using one of well-known mathematician methods (“Photon difference”, “Penfold-Leiss”, “Cook least structure”, “Tikhonov regularization”, etc.). All of them have been developed specially to obtain the effective photon spectrum (experiment apparatus function) looks like as quite enough monoenergetic function (line). It is important be pointed out that for this type of experiments apparatus function: 1) is constructed as enough narrow line independently of experiment conditions; 2) has complex (not ideal, for example, Gaussian line) shape that can produce some additional uncertainties in cross section shape, magnitude and position.

2) **QMA-experiments**<sup>1,2)</sup> are based on the process of producing annihilation photons with energy  $E_\gamma = E_{e^+} + 0.511 \text{ MeV}$  by fast positrons. Annihilation photons always are accompanied by positron bremsstrahlung and therefore three steps are needed (Fig. 1) - 1) measurement of yield  $Y_{e^+}(E_j, k)$  of reaction induced by photons from  $e^+$  both annihilation and bremsstrahlung; 2) measurement of yield  $Y_{e^-}(E_j, k)$  of reaction induced by photons from  $e^-$  bremsstrahlung; 3) measured yields subtraction for interpretation of difference obtained as reaction cross section “measured directly”

$$Y_{e^+}(E_j, k) - Y_{e^-}(E_j, k) = Y(E_j, k) \approx \sigma(k). \quad (2)$$

It must be pointed out that: 1) there is no beam of QMA-photons in reality: they are “arising” as two real spectra difference only; 2) apparatus function of each experiment is obtained individually because depends on both measurements (yields -  $Y_{e^+}$ ,  $Y_{e^-}$ ) conditions; 3)  $e^+$  annihilation occurs in many steps (bremsstrahlung production ( $e^- + A \rightarrow A + e^- + \gamma$ ); pairs production ( $\gamma + A \rightarrow A + e^- + e^+$ ); positron annihilation ( $e^+ + e^- \rightarrow 2\gamma$ )); therefore number of quasimonoenergetic photons, measured yields statistical accuracy, and hence their normalization accuracy are small.

The differences in apparatus functions (effective photon spectra) mean<sup>4-6)</sup> differences in experiment conditions and hence disagreements between obtained result interpretations. Very briefly – all photonuclear experiment results are not the cross sections really but the reaction yields only. Not real cross section can be obtained but its evaluation  $\sigma^{\text{eval}}(k) = \int F(E_{jm}, k) \sigma(k) dk$  differs from real cross section in accordance with that apparatus function (resolution function  $F(E_{jm}, k)$  differs from  $\delta$ -function.



Real energy resolution reflected the width of sum of positron annihilation and bremsstrahlung.

Theoretically calculated width of annihilation line in the photon spectrum.

It must be pointed out that QMA-experiment difference scheme gives to one possibility to exclude bremsstrahlung “tail” from effective photon spectrum but do not improve initially poor energy resolution – the special processing is needed for that.

**FIGURE 1.** Experimental yields of  $^{63}\text{Cu}(\gamma,n)^{62}\text{Cu}$  reaction<sup>3)</sup> (×), appropriate effective photon spectra (lines) and method of reduction apparatus function (dashed line):

- a) - final QMA-result -  $Y_{e^+}(E_j) - Y_{e^-}(E_j) = Y(E_j) \approx \sigma(k)$  (2) and difference between spectra of photons produced by positrons and electrons;
- b) - yield  $Y_{e^-}(E_j)$  and electron bremsstrahlung spectrum (Penfold-Leiss method apparatus function is presented additionally (+));
- c) - yield  $Y_{e^+}(E_j)$  and spectrum of photons produced by positrons (sum of bremsstrahlung and annihilation).

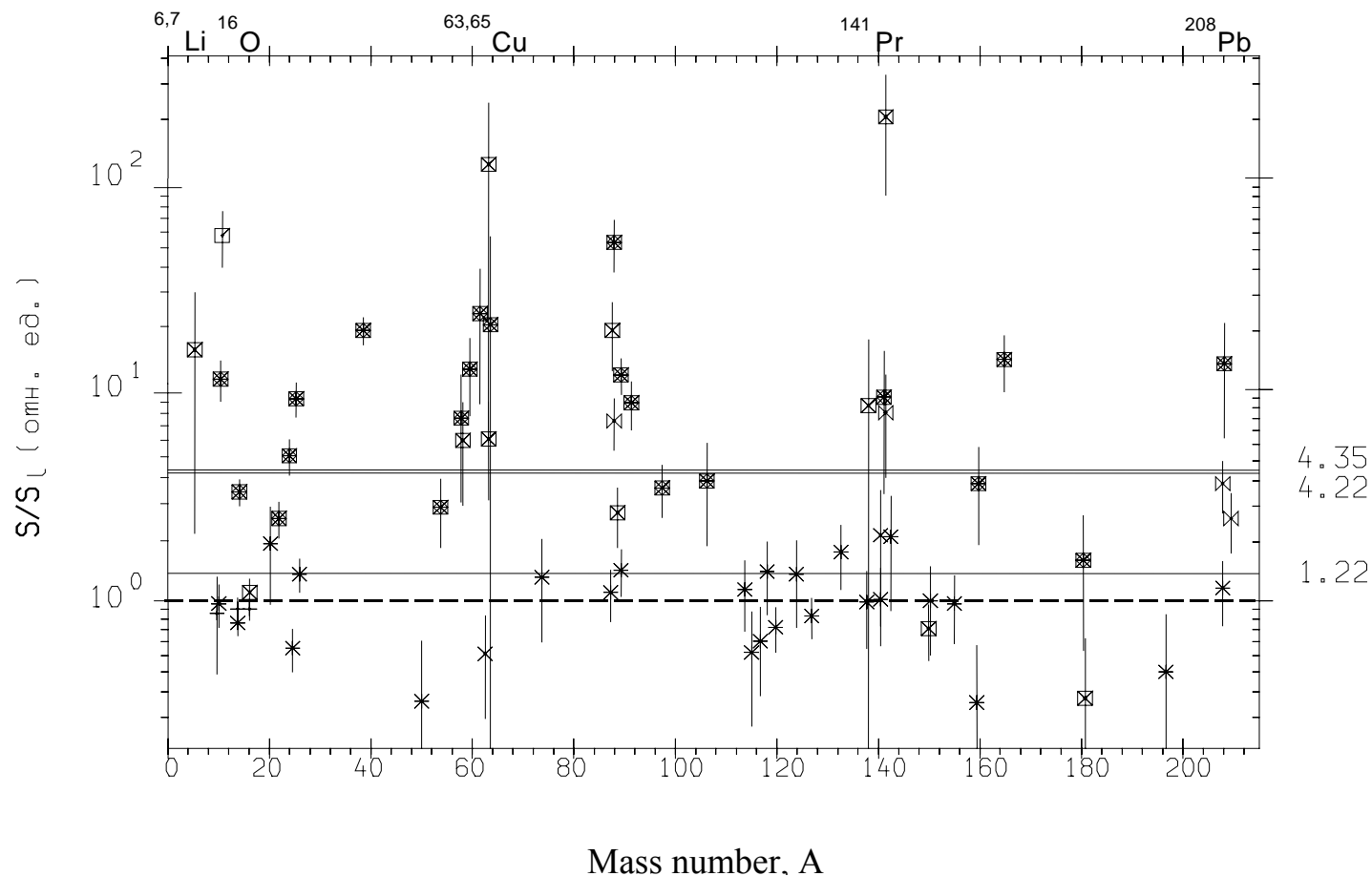
## Photonuclear Reaction Cross Section Systematical Disagreements

Numerical example of discrepancies concerned was obtained<sup>7)</sup> from the detailed comparison of  $^{18}\text{O}(\gamma, xn)$  reaction cross sections for obtained using BR-photons (University of Melbourne, Australia)<sup>7)</sup> and QMA-photons (Livermore)<sup>8)</sup>. Despite authors<sup>7)</sup> say about good agreement between both experiments data, their own analysis clearly shows that almost all resonances have larger amplitude and smaller width in BR- than in QMA-photon cross sections: amplitude ratios  $A_{\text{BR}}/A_{\text{QMA}}$  varied from 1.00 to 1.28 (mean value  $\langle A_{\text{BR}}/A_{\text{QMA}} \rangle = 1.17$  (17 % difference)), width ratios  $\Gamma_{\text{QMA}}/\Gamma_{\text{BR}}$  varied from 1.00 to 2.25 (mean value  $\langle \Gamma_{\text{QMA}}/\Gamma_{\text{BR}} \rangle = 1.35$  (35 % difference)).

The general systematic (more than 500 total photoneutron reaction  $(\gamma, xn)$  cross sections for nuclei from  $^3\text{H}$  to  $^{238}\text{U}$ )<sup>5)</sup> of disagreements concerned was analyzed for special parameter “structureness” described as whole the deviation of each reaction cross section from itself but significantly (with resolution about  $\Delta = 1$  MeV) smoothed:

$$S = \frac{1}{N} \sum_{i=1}^N \frac{(\sigma_i - \langle \sigma_i \rangle)^2}{\langle \langle \sigma \rangle \rangle^2}, \quad (3)$$

where  $\langle \sigma \rangle$  and  $\langle \langle \sigma \rangle \rangle$  are cross sections averaged for  $\Delta$  and D regions correspondingly. The ratios  $S/S_1$  were obtained specially where S were calculated for various laboratories data and  $S_1$  - for Livermore QMA-data (some other QMA-data were used also). Data are clearly separated into two groups: BR- ( $\langle S/S_1 \rangle = 4.35$ ) and QMA- ( $\langle S/S_1 \rangle = 1.22$ ). This means that in all QMA-laboratories estimation of energy resolution using annihilation line width (in may cases 250 – 400 keV, sometimes 500 keV, more rarely 150 – 300 keV) do not have real resolution: QMA-cross sections are over-smoothed. This is confirmed by the  $\langle S/S_1 \rangle = 4.22$  value for data obtained using tagged photons (apparatus function is close to Gaussian).



The ratios  $S/S_1$  are presented, where  $S$  were calculated for various laboratories data and  $S_1$  - for Livermore QMA-data (some other QMA-data were used also).

Systematics of cross section “structurenes”  $S/S_1$  ratios obtained for total photoneutron reaction cross section data:

- BR-data (⊠ - Moscow, ⊠ - Melbourne (Australia), ⊠ - other);
- QMA-data (\* - Saclay (France), + - Giessen (Germany), \* - other);
- TP-data (⊠ - Illinois (USA)).

Because QMA-cross section (2) in reality is only yield again (1), real cross section can be obtained<sup>4-6)</sup> only after additional processing using information on real apparatus function. Shortly method of reduction proposed is not the method of solving inverse ill-posed unfolding problem (1).

## Method of Reduction for Reaction Cross Section Evaluation

Method of reduction transforms the data obtained with some experimental apparatus function into those would have being measured by means of apparatus function of other (better, for example Gaussian line with exactly known energy resolution) quality: most reasonably achievable monoenergetic representation for reaction cross section from reaction yield.

The reaction yield (1) measured using the apparatus function A is written in operator form<sup>4-6)</sup>

$$y = A\sigma + v \quad (4)$$

and than using the simple transformation

$$Ry = R(A\sigma + v) = U\sigma + (RA - U)\sigma + Rv = \sigma^{\text{eval.}} \quad (5)$$

with the special operator

$$R = U(A^*\Sigma^{-1}A)^{-1}A^*\Sigma^{-1} \quad (6)$$

is transformed into the evaluated cross section]

$$\sigma^{\text{eval}} = Ry = U\sigma + Rv \quad (7)$$

with uncertainties

$$G = R\Sigma R^* = U(A^*\Sigma^{-1}A)^{-1}U^* \quad (8)$$

that represents “the measurement” of cross section using apparatus function U of needed quality.



**The Method of Reduction** calculating procedure includes several steps:

1) - The apparatus function  $U_0$  with which researcher needs to have a data, e.g. Gaussian with FWHM 10 keV (very high resolution), is ordered.

2) - The operator  $U$  which satisfies the condition  $RA - U$  (5) is picked out of the class of operators which provide the solution without a priori information, closest to  $U_0$

(in terms of Hilbert-Schmidt norm  $\|U - U_0\|_2^2 = \text{tr}((U - U_0)(U - U_0)^*)$ )

$$U = U_0 A^{-} A,$$

where the dash means pseudoinversion.

3) - If the difference between desired  $U_0$  and constructed  $U$  is too big then the FWHM of  $U_0$  should be increased, e.g. by 10 keV, and go to point 1).

4) - The Method of Reduction gives the least squares estimator of the vector (7)

$$\sigma^{\text{eval}} = Ry = U(\Sigma^{-1/2} A)^{-} \Sigma^{-1/2} Y$$

and covariance matrix (8) of the estimator.

5) - If the estimated statistical uncertainty is too big go to point 1).

The result obtained in this way has the most reasonably achievable resolution for all apparatus functions  $U$  having Gaussian-like lineshapes – monoenergetical representation for the reaction cross section investigated - and the statistical uncertainty as small as it satisfies the researcher.

The method described is not the method of “unfolding” of  $\sigma$  from  $Y$ . It just transforms the data obtained with apparatus function  $A$  to the form they would have being measured by means of apparatus function  $U$ : there is no need to solve the ill-posed problem (1).

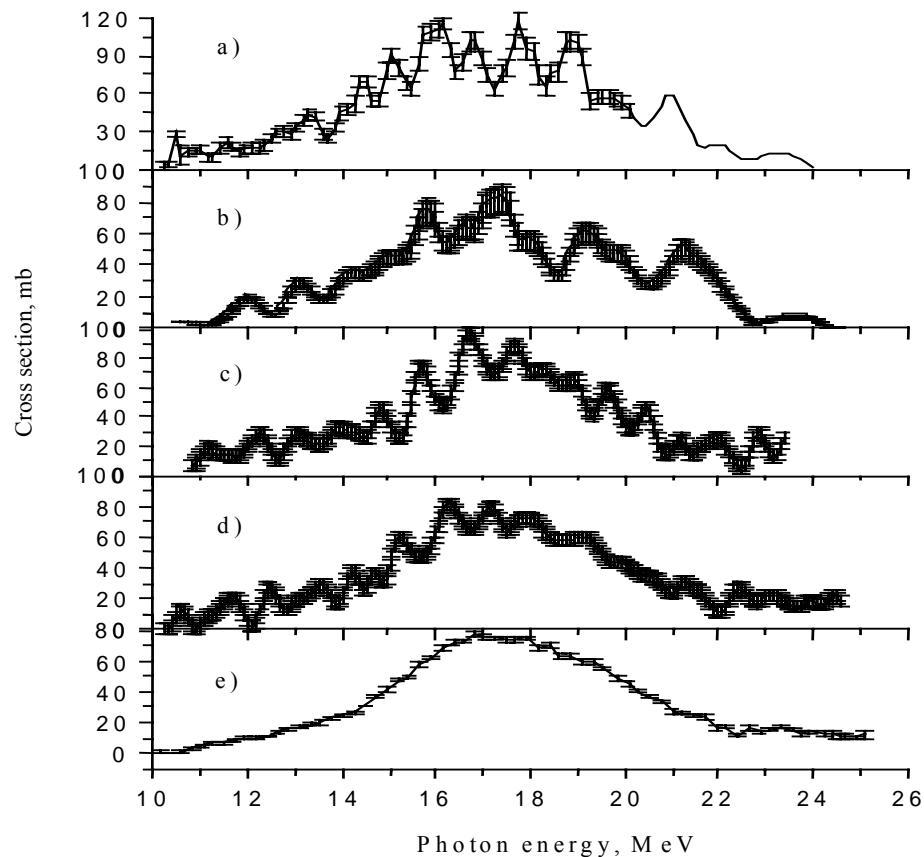
# QUASIMONOENERGETIC ANNIHILATION EXPERIMENT

## REAL ENERGY RESOLUTION ESTIMATION

Main result of processing using the method of reduction<sup>4-6)</sup> is that structure of QMA-cross sections became much more clear and closer to that obtained in BR-cross sections. As an example all three (two intermediate and one final (2)) typical QMA-results<sup>3)</sup> (used to obtain reaction cross section) are presented on Fig. 1 and cross section data obtained using method of reduction from all three of them are compared on Fig. 2 with the result of typical BR-experiment<sup>10)</sup> for the same energy resolution  $\Delta E = 210$  keV. Without special normalization and energy corrections positions of resonances differ not more than 200 keV and their amplitudes – not more than 15 % (see table below).

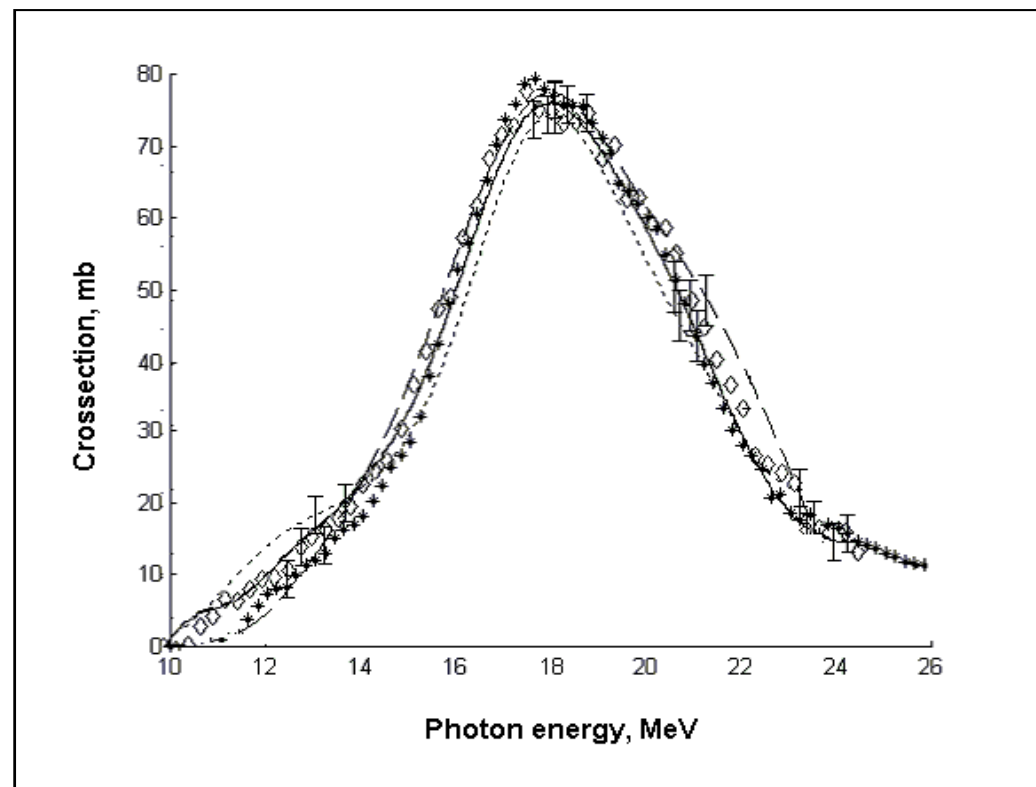
Inverse operation of evaluated cross sections from Figs. 2b, c, d of smoothing with the aim of their agreement with QMA-result gives<sup>6)</sup> that their energy resolutions are only  $\Delta E = 1.2 - 1.3$  MeV ( $\chi^2_{\min} = 0.03 - 0.05$ ) - 4 times worse than estimated width of annihilation line.

Analogous processing of  $^{197}\text{Au}(\gamma, xn)$  reaction data<sup>10)</sup> gives real  $\Delta E = 1.6$  MeV ( $\chi^2_{\min} = 0.11 - 0.18$ ) - 3 times worse than declared.



**FIGURE 2.**  $^{63}\text{Cu}(\gamma,n)^{62}\text{Cu}$  reaction cross sections:

- a) BR-experiment<sup>9)</sup> ( $\Delta E = 210$  keV);
- b) result of processing of QMA-yield (2)  $Y_e^-(E_j)^3$  (method of reduction for ( $\Delta E = 210$  keV);
- c) result of processing of QMA-yield (2)  $Y_e^+(E_j)^3$  (method of reduction for ( $\Delta E = 210$  keV);
- d) result of processing of QMA-yield difference (2)  $Y_{e^+}(E_j) - Y_{e^-}(E_j) = Y(E_j) \approx \sigma(k)$  (method of reduction for ( $\Delta E = 210$  keV);
- e) published<sup>3)</sup> QMA-yield difference (2)  $Y_{e^+}(E_j) - Y_{e^-}(E_j) = Y(E_j) \approx \sigma(k)$  ( $\Delta E \sim 200 - 400$  keV is declared).



**Comparison of  $^{63}\text{Cu}(\gamma,n)^{62}\text{Cu}$  reaction cross sections smoothed ( $\chi^2 = \min$ ) for better agreement with final result of QMA-experiment:**

- « $\diamond$ » BRA-experiment result;
- « $\square$ » processing of  $Y_{e^-}(E_j)$  - intermediate (2) QMA-experiment result;
- « $\star$ » processing of  $Y_{e^+}(E_j)$  - intermediate (2) QMA-experiment result;
- « $\text{---}$ » processing of  $Y_{e^+}(E_j) - Y_{e^-}(E_j)$  - final (2) QMA-experiment result;
- «\*»  $Y_{e^+}(E_j) - Y_{e^-}(E_j) = Y(E_j) \approx \sigma(k)$  - final (2) QMA-experiment result.

**Comparison of  $^{63}\text{Cu}(\gamma, n)^{62}\text{Cu}$  reaction cross sections  
obtained from various experiments data using method of reduction:  
energy positions (in MeV) and amplitudes (in mb) of structure resonances.**

Resonance energy	BR	QMA		
		$Y_{e^-}(E_j)$ (Fig. 2b)	$Y_{e^+}(E_j)$ (Fig. 2c)	$Y(E_j) = Y_{e^+}(E_j) - Y_{e^-}(E_j)$ (Fig. 2d)
$E_\gamma$ , M $\beta$ B	BR-cross section (Fig. 2a)			
15.8 – 16.1	112	61	57	53
16.7 – 16.9	95	55	70	70
17.7 – 18.0	104	68	64	68
19.3 – 19.5	94	51	50	52
21.7 – 22.0	50	45	20	28

Clear structure resonances are in good agreement with each other both in position and amplitude.

## IMPORTANT CONCLUSIONS

QMA-data are strongly over-smoothed (real energy resolution is several (3 – 4) times worse than declared one) in comparison with estimation based on calculated with of annihilation line in photon spectrum. The reason for that is quite simple. The difference procedure (2) used is oriented to bremsstrahlung “tail” subtracting but not for high resolution obtaining: both experimental results  $Y_{e^+}(E_j)$  and  $Y_{e^-}(E_j)$  have bad resolution determined by sum of large number of bremsstrahlung photons and small number of annihilation photons and therefore the resolution of their difference  $Y_{e^+} - Y_{e^-} = Y \approx \sigma$  must not be attributed only to annihilation line. QMA-data must be additionally reprocessed using, for example, the method of reduction ((4) – (8)), that reveals the GDR structure close to that obtained in BR-data.

Intermediate GDR structure (resonances with width  $\sim$  hundreds of keV) exists. BR-data look like preferable for GDR structure detailed study because QMA-data real energy resolution ( $\sim$  1.3 - 1.6 MeV) give not to one possibility to investigate physical effects produced structures with smaller width.

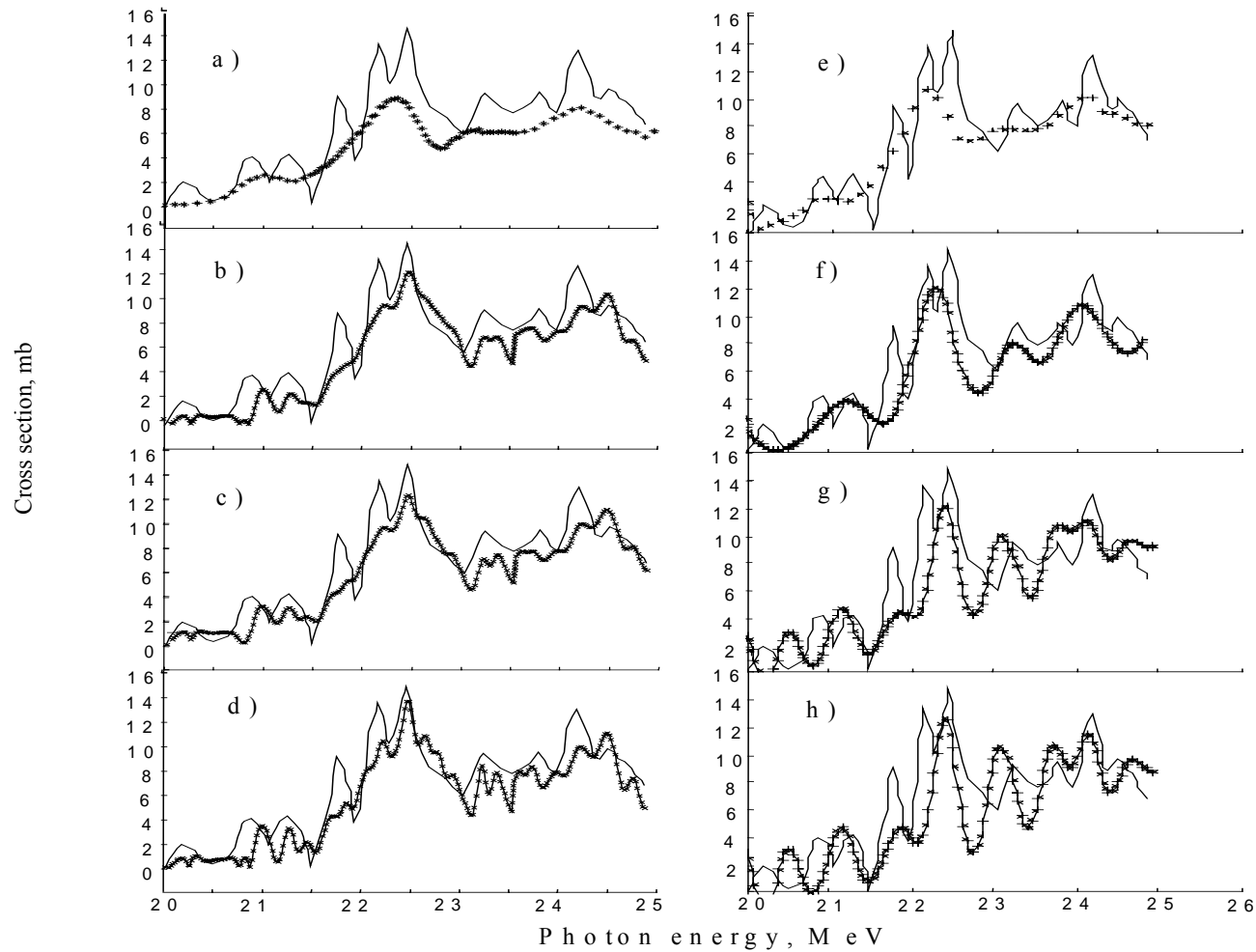
## ACKNOWLEDGMENTS

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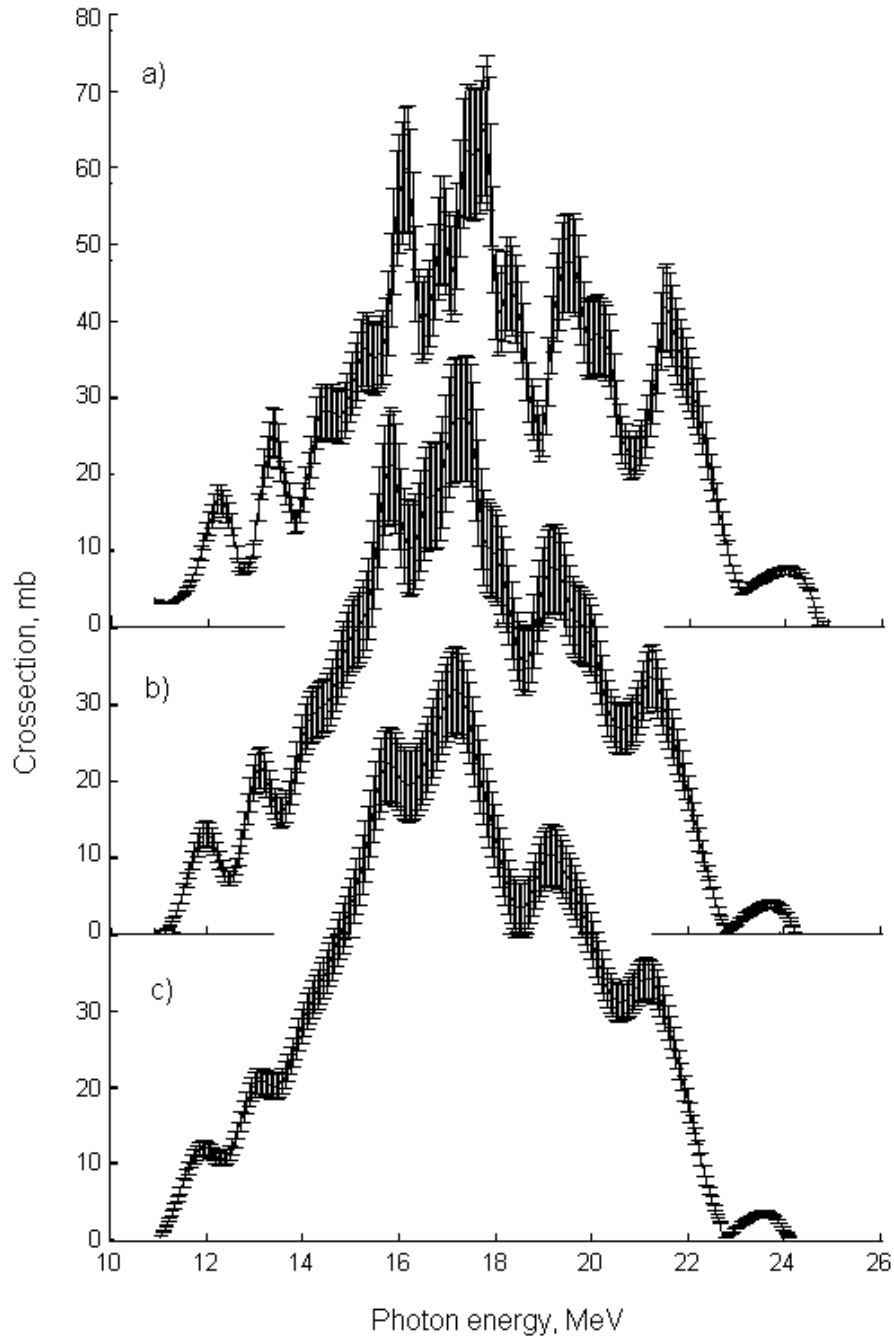
**QMA (Saclay) and BR (Moscow) - line- BR (Moscow) and QMA-Livermore**



**$^{16}\text{O}(\gamma x, n)$**

**Cross section structure dependence on real energy resolution obtained: more near are real energy resolution more similar are GDR structure features.**

**$^{63}\text{Cu}(\gamma,n)$  reaction QMA yield  $Y_e$ . (2) processing.**



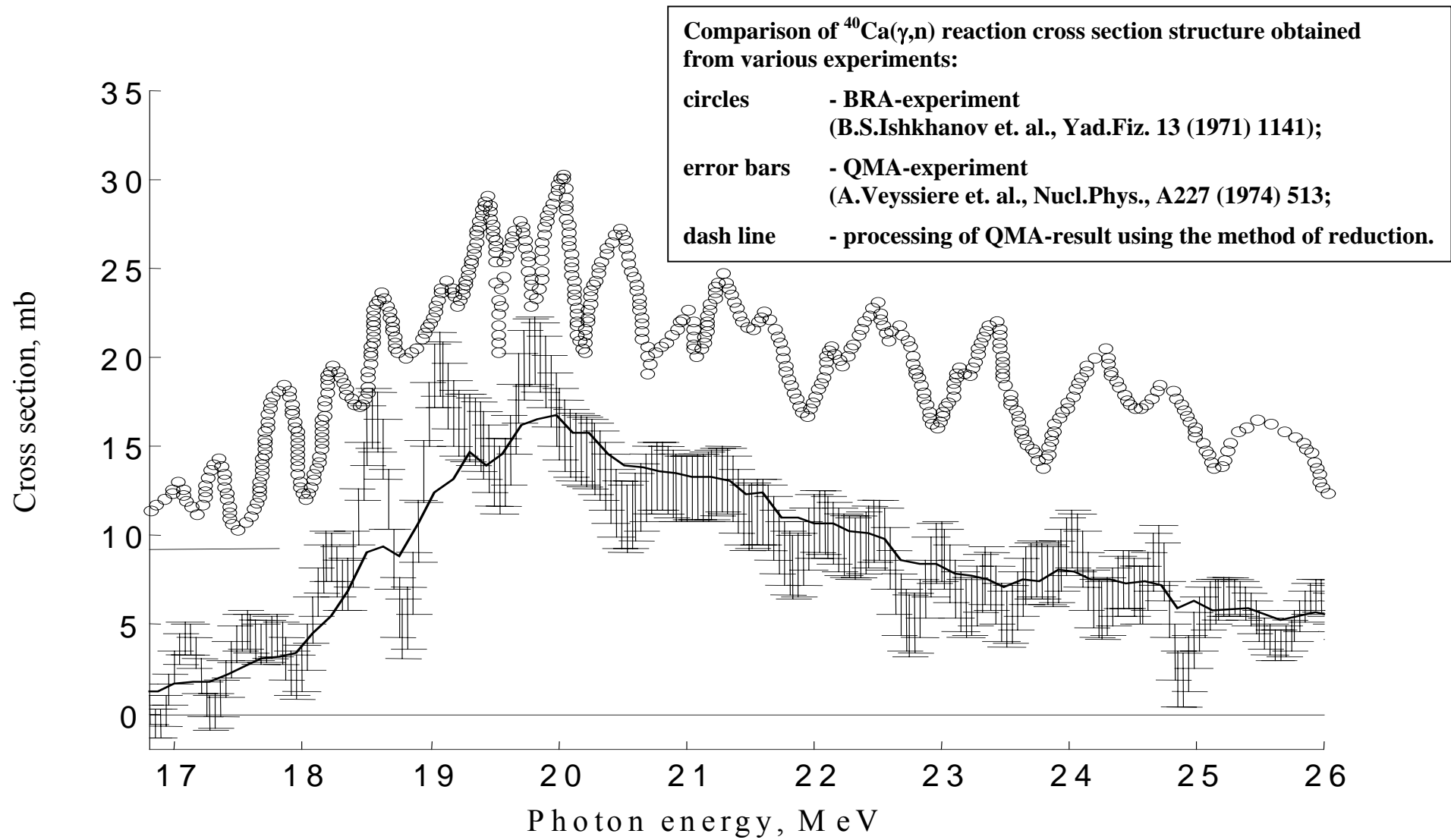
**Resolution 100 keV.**

**Resolution 250 keV.**

**Resolution 500 keV.**

**Cross section structure dependence on various real energy resolution: practically the same resonances (clearness depends on resolution obtained) are presented in both upper cross sections and the main of them are well separated at 500 keV also (from comparison with Fig 2e it is clear that the QMA-experiment resolution is more poorer).**

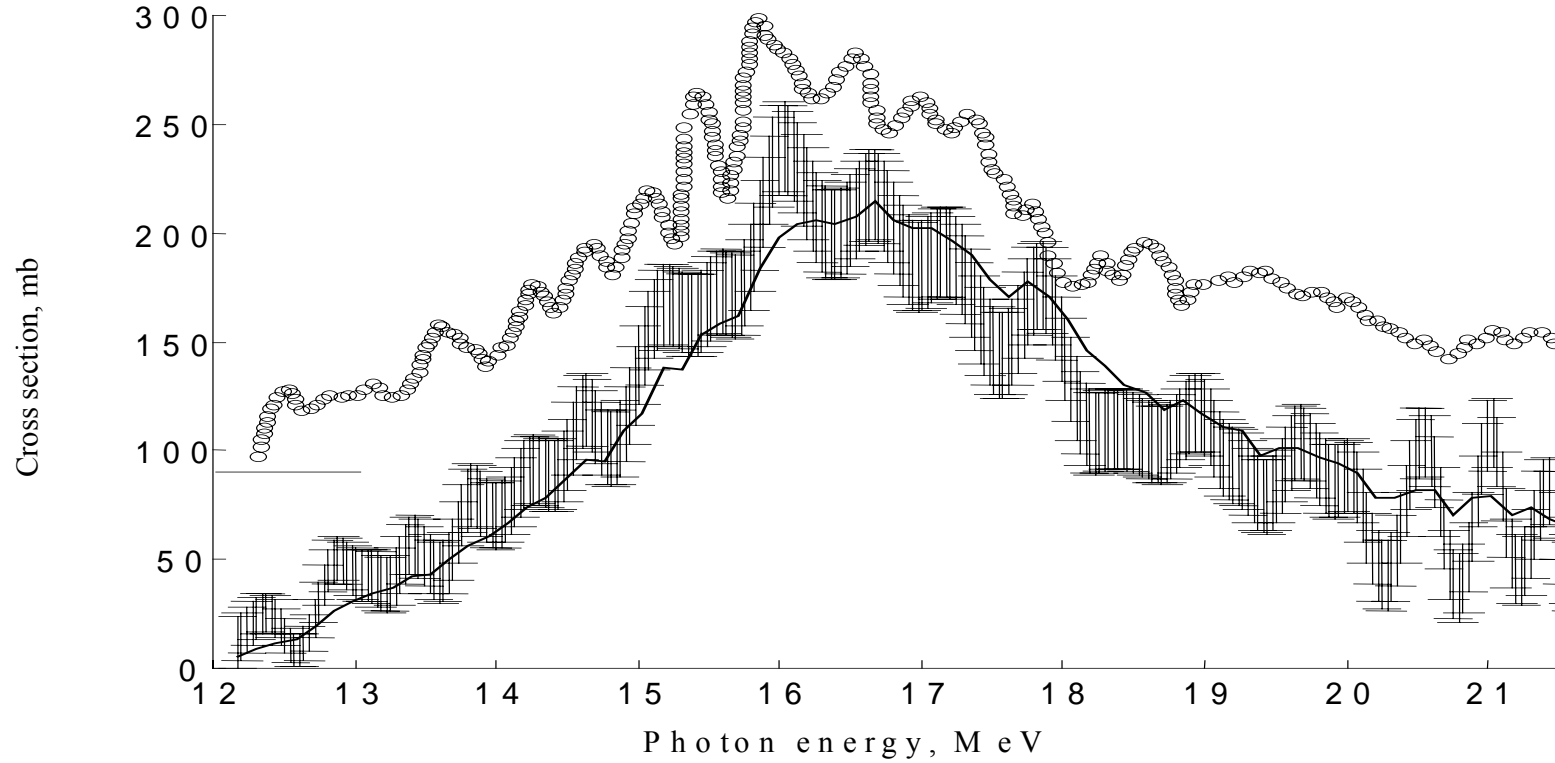




Experiment	Energy (MeV); *) - very small resonances												
<b>BRA</b>	17.1	17.8	18.2	18.6	19.3	19.9	20.4	21.2	22.5	23.3	24.2	24.7	25.5
<b>QMA processed</b>	17.1*)	17.8	18.2*)	18.7	19.1	19.8	20.3*)	21.0*)	22.4	23.2	24.1	24.6	25.3
<b>QMA</b>				18.7*)	19.2*)	20.0			22.3*)		24.1	24.7*)	25.4*)

**Comparison of  $^{90}\text{Zr}(\gamma, n)$  reaction cross section structure obtained from various experiments:**

- circles** - BRA-experiment (B.S.Ishkhanov et. al., Yad.Fiz. 14 (1971) 27);
- error bars** - QMA-experiment (A.Lepretre et. al., Nucl.Phys., A175 (1971) 609);
- dash line** - processing of QMA-result using the method of reduction.



Experiment	Energy (MeV); *) - very small resonances										
BRA	12.4	12.9	13.5	14.1	14.6	15.0	15.3	16.4	16.9	17.7	18.6
QMA processed	12.3	12.8	13.6*)	14.2	14.7*)	15.1	15.4*)	16.5	17.0*)	17.6	18.8
QMA						15.0*)		16.5	16.9*)	17.6	18.7